Int. J. Heat Mass Transfer. Vol. 6, pp. 1049–1052. Pergamon Press 1963. Printed in Great Britain.

CYLINDRICAL SECTIONS WITH UNIFORM DIFFUSE-RADIATION CHARACTERISTICS

MAX. A. HEASLET* and FRANKLYN B. FULLER†

National Aeronautics and Space Administration, Ames Research Center, Moffett Field, California

(*Received* 21 *January* 1963 *and in revised form* 7 *May* 1963)

Abstract—Calculations are made that determine the cross-sectional shapes of cylindrical or two-dimensional cavities that exhibit uniform radiation characteristics when the interior surface is a diffuse emitter. Immediate applications arise for concave cylindrical radiators or grooves on the surface of an opaque material when temperature is held fixed and uniform emission or heat flux is desired. Geometric interrelations between these and similar results for axially symmetric concavities are determined.

ANALYSIS

IT IS well known, by virtue of the radiation shape factor, that for three-dimensional shapes the interior of a spherical enclosure with an aperture yields uniform radiative heat flux for uniform surface temperature. In [1], Sparrow and Jonsson presented a summary analysis of the three-dimensional problem in a particularly concise form. Here, we show the two-dimensional analogue to their work along with some generalization. The geometry of the sectional shape differs considerably in detail from the circular arc that generates the spherical enclosure but, otherwise, a formal parallelism exists in the expressions involving energy fluxes (rate of energy transport per unit area as a function of emissivity, absorptivity, and geometry) in the two cases.

Consider an enclosure with an opening and continuous walls at constant temperature. The cavity receives no radiation from external sources. In the two-dimensional case we refer to the cross section of the enclosure, and elemental areas are thought of as possessing unit width normal to the plane of the section; the three-dimensional case refers to an arbitrary surface in space. Let the emission be diffuse so that Lambert's law applies. Absorption coefficient, α, and emission coefficient, ϵ, are given average values over the frequency range of the radiation and thus are parameters depending only on the nature of the surface material and its temperature. If the local radiant flux leaving the surface (the so-called radiosity) is denoted by the point function B, local incoming radiation flux is H, and local net heat flux is Q, we have

$$Q = B - H$$
$$B = \epsilon\sigma T^4 + (1 - \alpha)H$$
$$H = \int B \, \mathrm{d}F_{1-2} \qquad (1)$$

where $\mathrm{d}F_{1-2}$ is the angle factor between two surface elements. A concave enclosure is assumed so that the integration extends over the entire surface. The aperture plays a passive role and merely provides an avenue of escape of the net energy flux.

If B, T, Q, and H are uniform and if the contour of the surface is such that the integral of the differential shape factor over the surface is a constant K independent of position, we have

$$\frac{B}{\sigma T^4} = \frac{\epsilon}{\alpha}\left[1 - \frac{(1-\alpha)}{\epsilon}\,\frac{Q}{\sigma T^4}\right] = \frac{\epsilon}{1-(1-\alpha)K} \qquad (2)$$

where K depends on the geometry of the enclosure. Thus, in both two and three dimensions a single figure suffices to show either the radiosity or the heat flux. Fig. 1 shows $Q/\epsilon\sigma T^4$ as a function of α and K.

In three dimensions the desired geometrical

* Chief, theoretical branch.

† Research scientist.

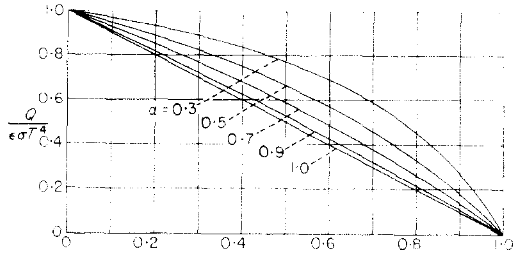


FIG. 1. Dimensionless heat flux as a function of absorption α and shape parameter K.

shape is a spherical cap with circular aperture. In this case, as noted by Sparrow and Jonsson,

$$K = \frac{1}{2}\frac{h}{R} = \frac{1 - \cos\theta}{2} = \frac{\eta^2}{1 + \eta^2} \tag{3}$$

where the notation is

- R, sphere radius;
- h, height of spherical cap;
- θ, half-vertex angle of cone with vertex at center of sphere and passing through rim of aperture;
- η, height of cap when the sphere geometry is scaled to give unit aperture radius.

In two dimensions, the sectional shapes cannot be identified so directly but they can be calculated by solving a differential equation. As in Fig. 2 let the aperture of the section of the cylinder have the length 2. Let P be an arbitrary point on the section, PN be normal to the section, R_l and R_r be distances from point P to $(-1, 0)$

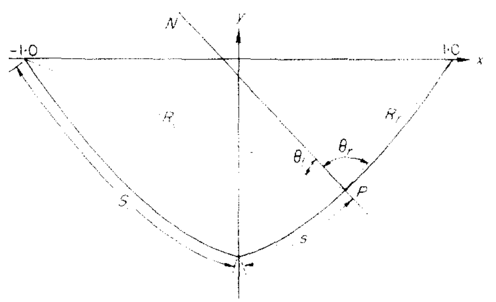


FIG. 2. Sketch showing co-ordinate system employed in deriving cylindrical section with uniform radiation characteristics.

and $(1, 0)$, respectively, and θ_l and θ_r be positive angles between the normal PN and the lines along which R_l and R_r are measured. Then, from [2], p. 19 *et seq.*, the integral of the shape factor yields the relation

$$K = \tfrac{1}{2}(2 - \sin\theta_l - \sin\theta_r). \tag{4}$$

Let Cartesian co-ordinates be used and the origin fixed so that the section is $y = y(x)$ with $y(x) = y(-x)$ and $y(-1) = y(1) = 0$. Equation (4) can be expressed in the differential form

$$\frac{x + 1 + yy'}{R_l} - \frac{x - 1 + yy'}{R_r} = 2(1 - K)(1 + y'^2)^{\frac{1}{2}} \tag{5}$$

where

$$R_r = [(x - 1)^2 + y^2]^{\frac{1}{2}}, \; R_l = [(x + 1)^2 + y^2]^{\frac{1}{2}},$$

and the prime denotes x-wise differentiation.

Equation (5) has the integral

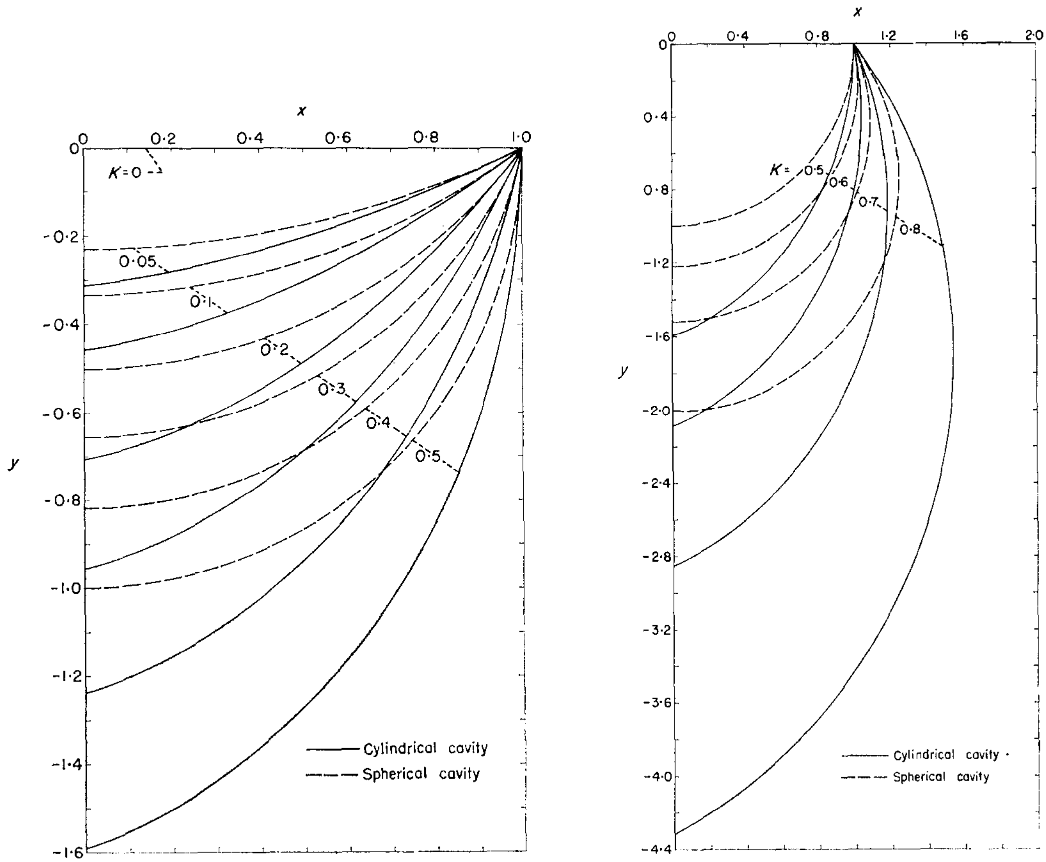
$$R_l - R_r = 2(1 - K)s \tag{6}$$

where s is arc length measured from the point of symmetry of the curve. Equation (6) is an interesting quantitative relation that must hold for three distances associated with any position of the point P. It could presumably be used to design a linkage system that could generate the desired curve; we have used it merely as a check on the end results attained by carrying out a further integration of equation (6). Figs. 3a and b show the sectional shapes drawn as full lines. These calculations were programmed for an electronic computer. The dashed curves superimposed on the figures show the spherical sections for the same values of K. These curves can be used in conjunction with Fig. 1.

Although the two-dimensional curves are symmetric, they differ from the circular-arc sections of the spherical enclosure in that a discontinuity in slope at the midpoint occurs. The slopes at the midpoint, $y'(0)$, are related to the local ordinate, $y(0)$, by the expression

$$1 = (1 - K)\{[1 + y(0)^2][1 + y'(0)^2]\}^{\frac{1}{2}}. \tag{7}$$

An additional relation of some interest



(a) 0·05 ≤ K ≤ 0·5. (b) 0·5 ≤ K ≤ 0·8.

FIG. 3. Two- and three-dimensional shapes with uniform radiation characteristics; K is geometric shape factor.

follows directly from equation (6) evaluated at $(-1, 0)$ or $(1, 0)$. This equality is

$$K = \frac{s_o - 1}{s_o} \quad (8)$$

where s_o is half the total arc length of the section. Reinterpretation of equations (2), (3), and (8) permits one to formulate the following:

Theorem: *For diffuse radiation, axially symmetric or long cylindrical enclosures with apertures can be constructed to yield uniform radiosity and heat flux at constant temperature. Their radiative characteristics are fixed by the universal function* $\epsilon/[1 - (1 - \alpha)K]$ *where* α *is absorption coefficient,* ϵ *is emission coefficient, and* K *is a geometric shape factor that is numerically equal to one minus the ratio of aperture area to concave surface area.*

REFERENCES

1. E. M. SPARROW and V. K. JONSSON, Absorption and emission characteristics of diffuse spherical enclosures, *J. Heat Transfer. Trans. ASME*, Ser. C, **84**, 188–189 (1962).
2. MAX JAKOB, *Heat Transfer*, Vol. II. Wiley, New York (1957).

Résumé—On a fait des calculs pour déterminer les formes des sections de cavités cylindriques ou bi-dimensionnelles qui présentent des caractéristiques de rayonnement uniforme quand la surface intérieure a une émission diffuse. Des applications immédiates se présentent pour des réflecteurs cylindriques concaves ou des rainures à la surface d'un matériau opaque quand, la température étant maintenue constante, on désire avoir une émission ou un flux de chaleur uniformes.

On a déterminé des relations géométriques entre ces résultats et des résultats semblables obtenus pour des concavités de révolution.

Zusammenfassung—Zur Bestimmung der Querschnittsformen zylindrischer oder zweidimensionaler Hohlräume mit einheitlicher Strahlungscharakteristik bei diffus strahlender Innenfläche, wurden Berechnungen durchgeführt. Eine unmittelbare Anwendung ergibt sich für konkav zylindrische Strahler oder Kerben in der Oberfläche undurchsichtiger Materialien mit vorgegebener Temperatur, wenn gleichmässige Abstrahlung oder konstanter Wärmefluss erwünscht ist. Geometrische Wechselbeziehungen zwischen diesen und ähnlichen Ergebnissen für achsialsymmetrische Hohlräume sind angegeben.